Application of the Multipoint Meshless FDM to Chosen Demanding Problems

Irena Jaworska

Institute for Computational Civil Engineering,
Cracow University of Technology
ul. Warszawska 24, 31-155 Cracow, Poland

Corresponding author: irena@L5.pk.edu.pl

Abstract. The paper focuses on alternative to finite element method approach based upon the new multipoint meshless finite difference method. The purpose of the work is to present the application of the multipoint meshless method to complex problems of engineering, as well as the selected results of the nonlinear and multiscale analysis. Moreover, several benefits of the proposed approach are highlighted, especially towards analysis of the nontrivial computational problems.

INTRODUCTION

The paper presents the application of the recently developed higher order multipoint meshless finite difference method to the demanding engineering problems, including nonlinear as well as multiscale analysis.

The meshless finite difference method (MFDM) belongs to the wide group of the so called meshless methods, which deal with the cloud of nodes only in contrast to the finite element method (FEM), where the computational mesh is required. These nodes can be arbitrarily irregularly distributed without any imposed structures, such as finite element, regular mesh, etc. As a consequence, local changes of the nodes discretization may be applied without difficulties. The local approximation of the unknown function is performed in the meshless methods rather around the nodes, than between them as it is in the FEM. The moving weighted least squares (MWLS) approximation is usually applied as the local one in the MFDM [1].

Recently, the MFDM was extended to the new multipont method [3], based on the old Collatz idea of the higher order FDM [2]. The multipoint method provides solution quality improvement and may be used to various types of engineering problems. The application of the multipoint method to the demanding problems, such as nonlinear ones and heterogeneous material analysis is presented here. The paper is illustrated by selected examples of multipoint numerical analysis.

MULTIPOINT MFDM AND ITS APPLICATION TO THE VARIOUS PROBLEMS

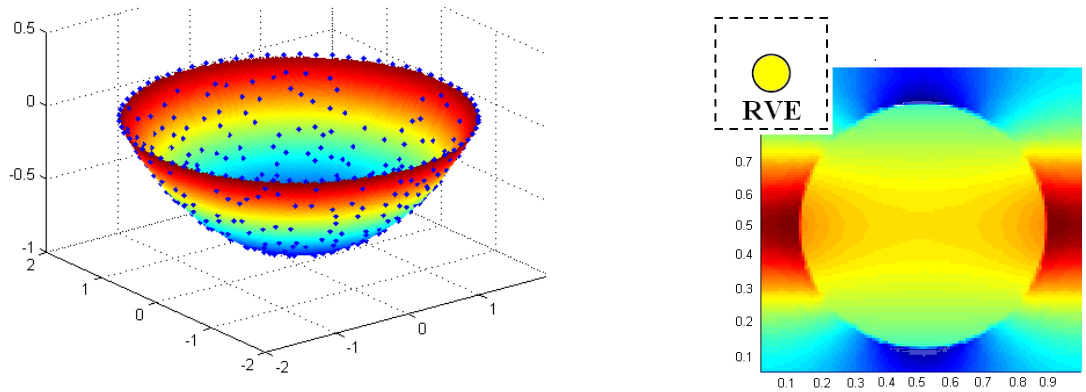
The method formulation [3], following the original Collatz [2] multipoint FD concept, has been modified and extended to the multipoint MFDM. The new multipoint method is based on the MWLS technique [1] instead of the polynomial interpolation proposed by Collatz. Moreover, arbitrarily irregularly distributed cloud of nodes, as well as local or various global formulation of a boundary value problem may be used here.

The idea of the multipoint approach is based on raising the approximation order of unknown function by using a combination of searched function values together with a combination of additional degrees of freedom at all stencil nodes. The known values of considered equation right hand side (specific case) or function unknown derivatives (general case [4]) may be applied as the additional d.o.f. This improves the solution quality without increasing the mesh density and uses the same stencil, as needed to generate FD operators in the standard MFDM approach [1].

Beside solution quality improvement, the essential features of the multipoint approach is its potentially wide range of applicability. The method is specially convenient for nonlinear analysis [5], because generation of all unknowns in terms of searched function values only. The MFD operators depend on discretization of the problem domain only. This fact is encouraging paying special attention to the calculation efficiency, especially for engineering tasks, where problem formulation may be changed, but the discretization is still the same. Also, due to higher order approximation, the method enables using decreased number of d.o.f.

NUMERICAL EXAMPLS

Several tests of application of the new multipoint method into nonlinear and multiscale analysis were carried out. Among other, the nonlinear engineering problem – deflection of the ideal membrane (Fig. 1a) and RVE-based analysis of heterogeneous elastic material with the inclusions spaced periodically [6, 7] (Fig. 1b) have been solved.



(a) (b)

FIGURE 1. (a) Numerical solution of ideal membrane deflection on irregular random mesh; (b) stress σ_{xx} calculated on the micro level

FINAL REMARKS

The paper presents the attempt of using the new multipont meshless FDM in analysis of more complex computational problems. The numerical results obtained so far are encouraging taking into account both their precision and efficiency. The approach is under current development. Further research is planned.

REFERENCES

1. J. Orkisz, "Finite Difference Method (Part III)", in *Handbook of Computational Solid Mechanics*, edited by M. Kleiber, (Springer-Verlag, Berlin, 1998), pp. 336-432.
2. L. Collatz, *Numerische Behandlung von Differential-gleichungen*, (Springer-Verlag, Berlin-Heidelberg, 1955).
3. I. Jaworska and J. Orkisz, Engineering Analysis with Boundary Elements **50**, 341-351 (2015).
4. I. Jaworska, Comput. Math. Appl., **66**(3), 238-249 (2013).
5. I. Jaworska and J. Orkisz, Engineering Analysis with Boundary Elements **92**, 231-243 (2018).
6. M. Geers, V. Kouznetsova and W. Brekelmans, Int. J. Multiscale Comput. Eng., **1**(4), 371–385 (2003).
7. I. Jaworska, Int. J. Multiscale Comput. Eng., **15**(4), 359–378 (2017).